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$$mk^2 \frac{d^2 \theta}{dt^2} = \frac{2a(1-c)PQ}{cP+Q} \dots\dots (8).$$

Integrating (8), supposing $d\theta/dt=0$, when $t=0$,

$$mk^2 \frac{d\theta}{dt} = \frac{2a(1-c)PQ}{cP+Q} t \dots\dots (9).$$

If there be no friction, as in (1) of the problem, $\mu=0$, $c=1$,

$$T=T' = \frac{2PQ}{P+Q} \dots\dots (10).$$

and (9) shows that there would be no rotation of the pulley.

85. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

A circular tube of radius a revolves uniformly about a vertical diameter with angular velocity $\sqrt{\frac{ng}{a}}$, and a particle is projected from its lowest point with such velocity that it can just reach the highest point; prove that the time of describing the first quadrant is $\sqrt{\frac{a}{(n+1)g}} \log (\sqrt{n+2} + \sqrt{n+1})$.

I. Solution by the PROPOSER.

Let $a\theta$ be the arc over which the particle has passed in any time t from the beginning of motion, R =reaction of the curve, g =the acceleration of gravity, and put $\sqrt{\frac{ng}{a}}=\omega$.

Resolving vertically and horizontally,

$$a \frac{d^2(\cos \theta)}{dt^2} = g - R \cos \theta \dots\dots (1), \quad a \frac{d^2(\sin \theta)}{dt^2} - \omega^2 a \sin \theta = -R \sin \theta \dots\dots (2).$$

Eliminating R ,

$$a \frac{d^2 \theta}{dt^2} - a \omega^2 \sin \theta \cos \theta = -g \sin \theta \dots\dots (3).$$

Integrating (3),

$$\frac{d^2 \theta}{dt^2} = \frac{2g}{a} \cos \theta - \omega^2 \cos^2 \theta + C \dots\dots (4).$$

When $\theta=0$, $\frac{d\theta}{dt} = \frac{4g}{a}$; $\therefore C = \frac{2g}{a} + \omega^2$, and (4) becomes

$$\frac{d\theta^2}{dt^2} = \frac{2g}{a} \cos\theta - \omega^2 \cos^2\theta + \frac{2g}{a} + \omega^2 \dots\dots (5).$$

$$\text{From (5), } dt = \frac{d\theta}{\sqrt{\left(\frac{2g}{a} \cos\theta - \omega^2 \cos^2\theta + \frac{2g}{a} + \omega^2\right)}} \dots\dots (6).$$

Let $\cos\theta = \frac{2-y}{3+y} \dots\dots (7)$, the limit of θ being 0 and $\frac{1}{2}\pi$, and of y , 2 and $-\frac{1}{2}$.

$$\text{From (7), } d\theta = \frac{\sqrt{5} dy}{(3+y)\sqrt{1+2y}} \dots\dots (8).$$

Substituting (7) and (8) in (6), and reducing,

$$dt = - \frac{dy}{\sqrt{\left[\frac{6g}{a} + \omega^2 + 2\left(\frac{7g}{a} + \omega^2\right)y + 4\left(\frac{g}{a} + \omega^2\right)y^2\right]}}$$

negative, since t increases as y decreases.

Integrating between the above limits for y ,

$$\begin{aligned} t &= \frac{1}{\sqrt{\left(\frac{g}{a} + \omega^2\right)}} \log \sqrt{\frac{\sqrt{\left(\frac{2g}{a} + \omega^2\right)} + \sqrt{\left(\frac{g}{a} + \omega^2\right)}}{\sqrt{\left(\frac{2g}{a} + \omega^2\right)} - \sqrt{\left(\frac{g}{a} + \omega^2\right)}}} \\ &= \sqrt{\left(\frac{a}{(n+1)g}\right)} \log[1 + \sqrt{(n+2)} + \sqrt{(n+1)}]. \end{aligned}$$

II. Solution by G. B. M. ZEER, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let $OB=z$, $BP=r$, $\angle BOP=\theta$, $PO=a$, ω =angular velocity, $s=a\omega$. The equations of motion are

$$d^2r/dt^2 - r\omega^2 = R(dz/ds), \quad d^2z/dt^2 = g - R(dr/ds),$$

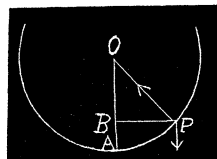
but $r = a \sin\theta$, $z = a \cos\theta$, $ds = ad\theta$.

$$\therefore a \cos\theta (d^2\theta/dt^2) - 2a\omega^2 \sin\theta = -R \sin\theta \dots\dots (1).$$

$$a \sin\theta (d^2\theta/dt^2) + a\omega^2 \cos\theta = -R \cos\theta - g \dots\dots (2).$$

Eliminating R between (1) and (2),

$$a(d^2\theta/dt^2) = a\omega^2 \sin\theta \cos\theta - g \sin\theta \dots\dots (3).$$



Integrating (3), $a(d\theta/dt)^2 = a\omega^2 \sin^2 \theta + 2g \cos \theta + C$.

Since $d\theta/dt=0$ when $\theta=\pi$, $C=2g$, also $\omega^2=ng/a$.

$\therefore a(d\theta/dt)^2 = ng \sin^2 \theta + 2g(1 + \cos \theta)$.

$$\therefore t = \sqrt{\frac{a}{g}} \int_0^{\frac{1}{2}\pi} \frac{d\theta}{\sqrt{(n+1)^2 - (1 - n \cos \theta)^2}} = \frac{1}{2} \sqrt{\frac{a}{g}} \int_0^{\frac{1}{2}\pi} \frac{\sec \frac{1}{2} \theta d\theta}{\sqrt{1 + n \sin^2 \frac{1}{2} \theta}}.$$

$$\therefore t = \frac{1}{2} \sqrt{\frac{a}{g}} \int_0^{\frac{1}{2}\pi} \frac{\sec^2 \frac{1}{2} \theta d\theta}{\sqrt{1 + (n+1) \tan^2 \frac{1}{2} \theta}}.$$

$$\text{Let } \tan \frac{1}{2} \theta = \sqrt{\frac{1}{n+1}} \tan \varphi.$$

$$\therefore t = \sqrt{\frac{a}{(n+1)g}} \int_0^{\tan^{-1} \sqrt{n+1}} \sec \varphi d\varphi = \frac{1}{2} \sqrt{\frac{a}{(n+1)g}} \log \left(\frac{1 + \sin \varphi}{1 - \sin \varphi} \right)_0^{\tan^{-1} \sqrt{n+1}}$$

$$\therefore t = \frac{1}{2} \sqrt{\frac{a}{(n+1)g}} \log \left(\frac{\sqrt{(n+2)} + \sqrt{(n+1)}}{\sqrt{(n+2)} - \sqrt{(n+1)}} \right) = \sqrt{\frac{a}{(n+1)g}} \log [\sqrt{n+2} + \sqrt{n+1}]$$

III. Solution by GEORGE E. DEAN, A. M., Professor of Mathematics, University of Missouri, School of Mines and Metallurgy, Rolla, Mo.

(1) Lagrange's equations for a point in space are

$$m\rho'' - m\rho(\theta'^2 + \sin^2 \theta \varphi'^2) = R,$$

$$m \frac{d}{dt}(\rho^2 \theta') - m\rho^2 \varphi'^2 \sin \theta \cos \theta = \rho \Theta,$$

$$m \frac{d}{dt}(\rho^2 \sin^2 \theta \varphi') = \rho \Phi \sin \theta;$$

where ρ , θ , φ are the polar coördinates, and R , Θ , Φ are the components of the impressed force along and normal to the radius vector. Besides these we have the geometrical equations $\rho=a$ and $\varphi'=c$. Also $R=mg \cos \theta$, $\Theta=mg \sin \theta$.

Replacing the first and last equations by the geometrical equations and eliminating ρ and φ' , we find

$$m \frac{d}{dt}(a^2 \theta') - ma^2 c^2 \sin \theta \cos \theta = mg a \sin \theta.$$

Removing factor ma , multiplying by $d\theta/dt$, and integrating,

$$a \left(\frac{d\theta}{dt} \right)^2 - ac^2 \sin^2 \theta = -2g \cos \theta + K.$$

When $\theta=0$, $d\theta/dt=0$; hence $K=2g$, and we have

$$a\left(\frac{d\theta}{dt}\right)^2 = 2g(1 - \cos\theta) + ac^2 \sin^2 \theta.$$

Putting $ac^2=ng$, and integrating between limits $\theta=\pi$, $\theta=\frac{1}{2}\pi$, we find

$$t = \sqrt{\frac{a}{(n+1)g}} \log(\sqrt{n+2} + \sqrt{n+1}).$$

(2) The energy of the particle relative to the tube is $\frac{1}{2}m a^2 \theta'^2$. This is due to two causes, gravity and rotation. Hence

$$\frac{1}{2}m a^2 \theta'^2 = amg(1 - \cos\theta) + \frac{1}{2}m a^2 \varphi'^2 \sin^2 \theta,$$

which is the same as the first integral second equation of the Lagrangian groups.

AVERAGE AND PROBABILITY.

74. Proposed by F. ANDEREGG, A. M., Professor of Mathematics, Oberlin College, Oberlin, Ohio.

From a point in the circumference of a circular field a projectile is thrown at random with a given velocity which is such that the diameter of the field is equal to the greatest range of the projectile. Find the chance of its falling into the field. [From Byerly's *Integral Calculus*, page 209.]

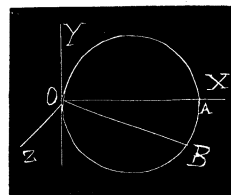
I. Solution by the PROPOSER.

It is easily seen that if α , the angle of projection, has a value from $\frac{1}{2}\pi - \frac{1}{2}\phi$ to $\frac{1}{2}\pi + \frac{1}{2}\phi$, the projectile will fall beyond B.

The unfavorable chance is

$$\frac{1}{\pi} \int_0^{\frac{1}{2}\pi} \int_{\frac{1}{2}\pi - \frac{1}{2}\phi}^{\frac{1}{2}\pi + \frac{1}{2}\phi} \cos \alpha \, d\alpha \, d\phi = \frac{2}{\pi} (\sqrt{2} - 1).$$

Since half of the projectiles will fall on the left of the YZ plane, the favorable chance is $\frac{1}{2} - (2/\pi)(\sqrt{2} - 1)$.



II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let φ = angle of projection, $OA=2a$, $\angle AOB=\theta$.

$\therefore$ Range $= (v^2/2g) \sin 2\varphi = OB = 2a \cos \theta$, when the projectile falls within the field.

The range is greatest when $\theta=45^\circ$, and then according to the conditions of the problem $v^2/2g=2a$.

$$\therefore 2a \sin 2\varphi = 2a \cos \theta, \text{ or } \sin 2\varphi = \cos \theta.$$

$$\therefore \sin \varphi = \frac{1}{2}(1 + \cos \theta)^{\frac{1}{2}} \pm \frac{1}{2}(1 - \cos \theta)^{\frac{1}{2}}.$$